MATHEMATICS (ELECTIVE) 2

1. STANDARD OF THE PAPER

The Chief Examiner for Mathematics (Elective) reported that the standard of the paper compared favourably with that of the previous years.

Candidates' performance was a notch higher than the previous year.

2. SUMMARY OF CANDIDATES' STRENGTHS

The Chief Examiner for Mathematics (Elective) listed the strengths of candidates as follows; ability to:

(1) find identity element and inverse of a binary operation;
(2) find composite functions;
(3) resolve forces into components;
(4) find the magnitude of the resultant of various forces;
(5) resolve functions into partial fractions;
(6) solve problems in probability;
(7) identify nature of roots of a quadratic function.

3. SUMMARY OF CANDIDATES' WEAKNESSES

The Chief Examiner for Mathematics (Elective) listed some of the weakness of candidates as difficulty in:

(1) drawing a Histogram;
(2) calculating of Spearman's Rank Correlation Coefficient;
(3) solving problems involving both Arithmetic/Geometric Progression;
(4) using properties of the scalar (Dot) Product to solve problems;
(5) drawing accurate tree-body diagrams to represent problems in mathematics;
(6) using of cosine/sine ranks appropriately in a triangle.

4. SUGGESTED REMEDIES FOR THE WEAKNESSES

(1) Teachers should give equal attention to all topics in the syllabus rather than specializing on some topics.
(2) Candidates should be given more exercises for them to have good command of the topics in the syllabus.
(3) Candidates should be encouraged to show all steps used in solving a problem clearly without jumping the steps in arriving at the solution.
(4) In general, teachers should pay attention to the weaknesses outlined above.

5. **DETAILED COMMENTS**

QUESTION 1

A binary operation Δ is defined on the set of real numbers, *R*, by *x* Δ *y* = *x* + *y* + 10. Find the:

(a) identity element;

(b) inverses of 3 and – 5 under Δ

Candidates were required to find the identity element for the binary operation $x \Delta y = x + y + 10$ and to find the inverse of 3 and -5 under the operation. The question was attempted by most candidates and was well handled. Thus, a good number of them were able to find the identity element and the inverse.

Candidates were required to solve (a) and (b) as follows:

(a)
$$x\Delta y = x + y + 10$$
$$x\Delta e = x + e + 10 = x$$
$$x + e + 10 = x$$
$$e = -10$$

(b)
$$3 \Delta x^{-1} = 3 + x^{-1} + 10 = -10$$
$$3 + x^{-1} + 10 = -10 \Rightarrow x^{-1} = -23$$
$$-5\Delta x^{-1} = -5 + x^{-1} + 10 = -10$$
$$-5 + x^{-1} + 10 = -10 \Rightarrow x^{-1} = -15$$

QUESTION 2

Evaluate $\int_2^4 \left(\frac{x^3+3}{x^2}\right)dx$.

Candidates were required to evaluate the integral $\int_2^4 \left(\frac{x^3+3}{x^2}\right)dx$. The question was attempted by most of the candidates and performance was good. Few candidates could not divide through by x^2 to obtain $x + 3x^{-2}$ before integrating. Such candidates found it difficult to integrate.

Candidates were required to solve question 2 as follows:

$$\int_2^4 \frac{x^3+3}{x^2}\,dx = \int_2^4 (x + 3x^{-2})\,dx$$
$$= \left[\frac{x^2}{2} + \frac{3x^{-1}}{-1}\right]_2^4$$
$$= \left[\frac{x^2}{2} - \frac{3}{x}\right]_2^4$$

$$= \left(8 - \frac{3}{4}\right) - \left(2 - \frac{3}{2}\right)$$
$$= 7.25 - 0.5$$
$$= 6.75$$

<u>QUESTION 3</u>

(a) **Two functions f and g are defined on the set of real numbers, *R* by *f*: $x \longrightarrow x^2 - 1$ and $g : x \rightarrow x+2$. Find $f \circ g(-2)$**

(b) **A bus has 6 seats and there are 8 passengers. In how many ways can the bus be filled?**

The question was in two parts, (a) and (b).

The question was attempted by most of the candidates. The part (a) was answered very well; the second part posed a problem for some of them. That part (b) required the use of permutation instead of combination. Candidates were required to answer the question as follows:

(a)
$$f: x \to x^2 - 1, \; g: x \to x + 2$$
$$f \; o \; g = (x+2)^2 - 1$$
$$= x^2 + 4x + 4 - 1$$
$$= x^2 + 4x + 3$$
$$f \; o \; g(-2) = (-2)^2 + 4(-2) + 3$$
$$= 4 - 8 + 3$$
$$= -1$$

(b)
$$8_{P_6} = \frac{8!}{(8-6)!} = \frac{8!}{2!}$$
$$= 20160 ways$$

<u>QUESTION 4</u>

Express $\frac{1}{x^2 - 16}$ in partial fractions.

A good number of candidates attempted the question and were able to resolve the expression in partial fractions.

Candidates were required to answer the question as follows:
$$\frac{1}{x^2 - 16} = \frac{1}{(x-4)(x+4)}$$

$$\frac{1}{(x-4)(x+4)} \equiv \frac{P}{x-4} + \frac{Q}{x+4}$$

$$1 = P(x+4) + Q(x-4)$$

$$put\ x = 4$$

$$1 = 8P$$

$$P = \tfrac{1}{8}$$

$$Next\ put\ x = -4, 1 = -8Q$$

$$\therefore Q = -\frac{1}{8}$$

$$\frac{1}{x^2-16} \equiv \frac{\frac{1}{8}}{x-4} + \frac{-\frac{1}{8}}{x+4}$$

$$\equiv \tfrac{1}{8(x-4)} - \tfrac{1}{8(x+4)}$$

<u>QUESTION 5</u>

The table shows the marks scored by some students in a class test.

Marks	**11 – 14**	**15 - 18**	**19 – 22**	**23 - 26**	**27 - 30**	**31 - 34**	**35 - 38**
No. of students	**4**	**5**	**18**	**31**	**25**	**14**	**3**

(a) **Draw a histogram for the distribution.**

(b) **Use the histogram to estimate the modal score, correct to one decimal place.**

Candidates were required to draw a histogram from a frequency table and use the histogram to estimate the modal mark. The question was attempted by most of the candidates. They were able to construct the table but could not draw the histogram correctly by using either the midpoints or the class boundaries. They could not show the exact positions of the class boundaries or the midpoints. Candidates' performance was generally very poor.

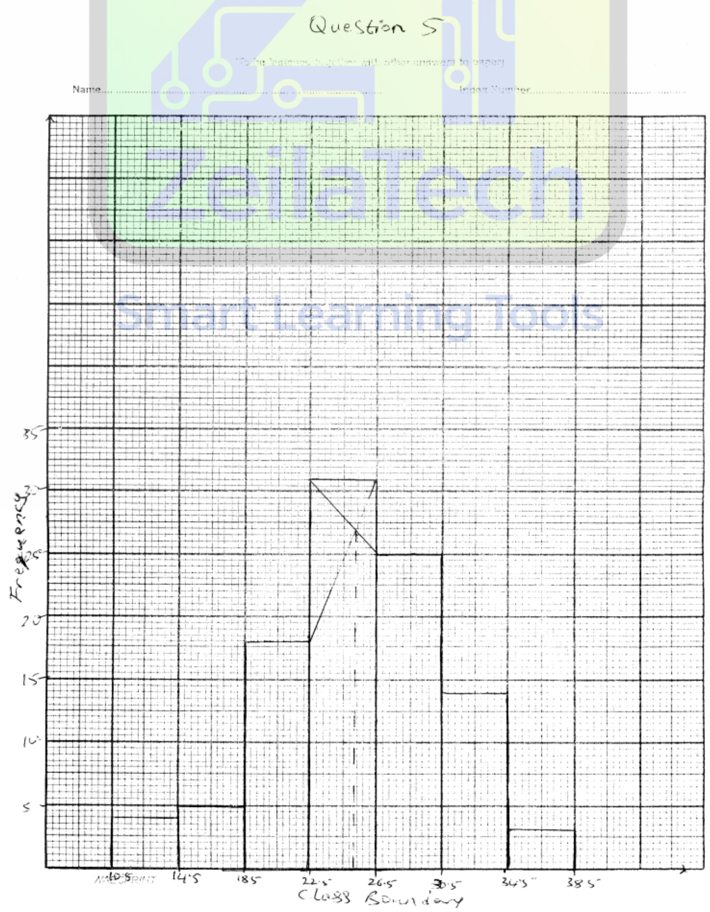
Candidates were required to answer the question as follows:

(a)

Class Boundary	f
10.5 – 14.5	4
14.5 – 18.5	5
18.5 – 22.5	18
22.5 –26.5	31
26.5 – 30.5	25
30.5 – 34.5	14
34.5 – 38.5	3

(b) From the graph

Mode = 25.2 ± 0.1



QUESTION 6

A bag contains 10 black and 5 yellow identical balls. Two balls are picked at random from the bag one after the other without replacement. Calculate the probability that they are:

(a) both black;

(b) of the same colour.

Candidates were required to find the probability of picking two balls at random from a bag without replacement. Almost all candidates who attempted the question got the answer correct. Most candidates used combination to solve the problem.

Candidates were required to answer the question as follows:

(a) $black = 10, yellow = 5, total = 15$

$$\text{P(both black)}\frac{\binom{10}{2}\binom{5}{0}}{\binom{15}{2}} = \frac{45\times1}{105}$$

$$= \frac{45}{105}$$

$$= \frac{3}{7} \text{ or } 0.4286$$

(b) $$\text{P(the same colour)} = \frac{\binom{10}{2}\binom{5}{0} + \binom{10}{0}\binom{5}{2}}{\binom{15}{2}}$$

$$= \frac{(45\times1)+(1\times10)}{105}$$

$$= \frac{45+10}{105}$$

$$= \frac{55}{105}$$

$$= \frac{11}{21} \text{ or } 0.5238$$

QUESTION 7

Force F_1 (24 N, 120^0), F_2(18N, 240^0) and F_3(12N,300^0) act at a point. Find, correct to two decimal places, the magnitude of their resultant force.

Candidates were required to find the magnitude of the resultant of three forces acting at a point. A good number of candidates attempted this question and it was well answered by most of them. They were able to resolve the forces into components, simplified and obtained the magnitude of the forces. Few candidates interchanged the *x* and *y* components of the forces which led to loss of marks.

Candidates were required to answer the question as follows:

$$F_R = F_1 + F_2 + F_3$$

$$F_R = \begin{pmatrix} 24sin120^0 \\ 24cos120^0 \end{pmatrix} + \begin{pmatrix} 18sin240^0 \\ 18cos240^0 \end{pmatrix} + \begin{pmatrix} 12sin300^0 \\ 12cos300^0 \end{pmatrix}$$

$$= \begin{pmatrix} 12\sqrt{3} \\ -12 \end{pmatrix} + \begin{pmatrix} -9\sqrt{3} \\ -9 \end{pmatrix} + \begin{pmatrix} -6\sqrt{3} \\ 6 \end{pmatrix}$$

$$= \begin{pmatrix} -3\sqrt{3} \\ -15 \end{pmatrix}$$

$$|F_R| = \sqrt{\left(3\sqrt{3}\right)^2 + (15)^2} = \sqrt{27 + 225} = \sqrt{252}$$

$$= 15.87N$$

QUESTION 8

The vectors p, q and r are mutually perpendicular with $|q|$ = 3 and $|r|$ = $\sqrt{5.4}$. If the vectors X = 3p + 5q + 7r and Y = 2p + 3q – 5r are perpendicular, find $|p|$.

Candidates were required to find one of three mutually perpendicular vectors. In the process they were required to apply the properties of the dot product. Most of the candidates avoided the question. Very few of them who attempted gave answers which showed that they have little knowledge of dot product of (perpendicular) vectors which the question required. The question was poorly answered.

Candidates were required to answer the question as follows:

$$\underline{X} = 3p = 5q + 7r \text{ and } \underline{Y} = 2p + 3q - 5r$$

$$\underline{X} \cdot \underline{Y} = \left(3\underline{p} + 5\underline{q} + 7\underline{r}\right) \cdot \left(2\underline{p} + 3\underline{q} - 5\underline{r}\right) = 0$$

$$= 6\underline{p} \cdot \underline{p} + 9\underline{p} \cdot \underline{q} - 15\underline{p} \cdot \underline{r} + 10\underline{q} \cdot \underline{p} + 15\underline{q} \cdot \underline{p}$$

$$-25\underline{q} \cdot \underline{r} + 14\underline{r} \cdot \underline{p} + 21\underline{r} \cdot \underline{q} - 35\underline{r} \cdot \underline{r} = 0$$

$$6|p|^2 + 15|q^2| - 35|r^2| = 0$$

$$6|p^2| = 54$$

$$\therefore |p| = 3$$

QUESTION 9

(a) **If $(p + 1)x^2 + 4px + (2p + 3) = 0$ has equal roots, find the integral value of p.**

(b) **Solve for x and y in the equations: $\log(x - 1) + 2\log y = 2\log 3$;**
$\log x + \log y = \log 6$

Most of the candidates attempted this question and scored good marks for the part (a). A good number of the candidates could not remove the logs in the part (b).

Candidates were required to answer the question as follows:

(a)

$$(p+1)x^2 + 4px + (2p+3) = 0$$

$$\text{For equal roots: } b^2 - 4ac = 0$$

$$16p^2 - 4(p+1)(2p+3) = 0$$

$$8p^2 - 20p - 12 = 0$$

$$2p^2 - 5p - 3 = 0$$

$$(p-3)(2p+1) = 0$$

$$p = -\frac{1}{2} \text{ or } p = 3$$

$$\therefore p = 3$$

(b)

$$\log(x-1) + 2\log y = 2\log 3$$

$$\log(x-1) + \log y^2 = \log 3^2$$

$$y^2(x-1) = 9 \ \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

$$\log x + \log y = \log 6$$

$$xy = 6 \ \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

$$x = \frac{6}{y}$$

$$y^2\left(\frac{6}{y} - 1\right) = 9$$

$$y^2 - 6y + 9 = 0$$

$$y^2 - 3y - 3y + 9 = 0$$

$$y(y-3) - 3(y-3) = 0$$

$$(y-3)(y-3) = 0$$

$$y = 3$$

$$x = \frac{6}{3}$$

$$\therefore x = 2$$

QUESTION 10

(a) Differentiate $y = \frac{3x}{1+x^2}$ with respect to x.

(b) Find the equation of the circle that passes through (2,3), (4,2) and (1,11).

(a) The part (a) was attempted by most of the candidates. Some candidates could not use the quotient rule correctly.

(b) The part (b) was correctly answered by most of the candidates. Candidates did the correct substitution to generate the three equations but were unable to solve the three equations well.

Candidates were required to answer the question as follows:

(a) $$y = \frac{3x}{1+x^2}$$

$$\frac{dy}{dx} = \frac{(1+x^2)\cdot 3 - 3x\cdot(2x)}{(1+x^2)^2}.$$

$$= \frac{3+3x^2-6x^2}{(1+x^2)^2}$$

$$= \frac{3-3x^2}{(1+x^2)^2}$$

$$= \frac{3(1-x^2)}{(1+x^2)^2}$$

(b)

$$x^2+y^2+2gx+2fy+c=0$$

At (2,3), $4+9+4g+6f+c=0$

$4g+6f+c=-13$ ……(i)

At (4,2), $16+4+8g+4f+c=0$

$8g+4f+c=-20$……(ii)

At (1,11), $1+121+2g+22f+c=0$

$2g+22f+c=-122$.....(iii)

(ii) – (i), $4g+2f=-7$……..(iv)

(ii) – (iii), $-6g+18f=-102$….(v)

$g = \frac{-11}{2}$

$f=-15$

$c=54$

$$x^2+y^2+2\left(-\frac{11}{2}\right)x+2\left(-\frac{15}{2}\right)y+6=0$$

$$x^2+y^2-11x-15y+54=0$$

QUESTION 11

When the terms of a Geometric Progression (G.P.) with common ratio r = 2 is added to the corresponding terms of an Arithmetic Progression (A.P.), a new sequence is formed. If the first terms of the G. P. and A. P. are the same and the first three terms of the new sequence are 3, 7 and 11 respectively, find the n^{th} term of the new sequence.

The question was on *AP* and *GP* was answered by most of the candidates. A few of the candidates who attempted it could not make any headway. Candidates were required to answer the question as follows:

$G.\,P = a, 2a, 4a$

$A.\,P = a, a+d, a+2d.$

$G.\,P + A.\,P = 2a + 3a+d + 5a+2d$

$2a = 3$

$a = \frac{3}{2}$

$3a + d = 7$

$d = \frac{7}{4}$

For new sequence

$a = 3, d = 4$

$a_n = 3 + 4(n-1)$

$= 4n - 1$

QUESTION 12

(a) **The probabilities that Golu, Kofi and Barry will win a competition are $\frac{1}{3}, \frac{2}{5}$ and $\frac{1}{2}$ respectively. Find the probability that only two of them wins the competition**

(b) **Ten eggs are picked successively with replacement from a lot containing 10% defective eggs. Find the probability that at least two are defective.**

Candidates were required to find the probabilities of events in different statement and cases. The part (a) was well answered by most of the candidates.

The part (b) however, which required the binomial probability was poorly answered.

Candidates were required to answer the question as follows:

(a)

$$P(G) = \frac{1}{3}, P(K) = \frac{2}{5}, P(B) = \frac{1}{2}$$

$$P(G^1) = \frac{2}{3}, P(K^1) = \frac{3}{5}, P(B^1) = \frac{1}{2}$$

G wins, K wins, B lose $= \frac{1}{3} \times \frac{2}{5} \times \frac{1}{2} = \frac{1}{15}$

G wins, K lose, B wins $= \frac{1}{3} \times \frac{3}{5} \times \frac{1}{2} = \frac{1}{10}$

G lose, K wins, B wins $= \frac{2}{3} \times \frac{2}{5} \times \frac{1}{2} = \frac{2}{15}$

P(only two wins) $= \frac{1}{15} + \frac{1}{10} + \frac{2}{15} = \frac{9}{30}$

$= \frac{3}{10}$ or 0.3

$$n = 10, \quad p = \frac{1}{10}, \quad q = \frac{9}{10}$$

(b)

$$P(X \geq 2) = 1 - P(X < 2)$$
$$= 1 - [P(X = 0) + P(X = 1)]$$
$$P(X = 0) = \binom{10}{0}\left(\frac{1}{10}\right)^0 \left(\frac{9}{10}\right)^{10} = 0.3487$$
$$P(X = 1) = \binom{10}{1}\left(\frac{1}{10}\right)^1 \left(\frac{9}{10}\right)^{9} = 0.3874$$
$$P(X \geq 2) = 1 - (0.3487 + 0.3874))$$
$$= 1 - 0.7361$$
$$\therefore P(X \geq 2) = 0.2639$$

QUESTION 13

The marks awarded by three examiners are given in the table:

Candidate	**A**	**B**	**C**	**D**	**E**	**F**	**G**	**H**	**I**	**J**
***Examiner* I**	**90**	**88**	**71**	**65**	**32**	**72**	**70**	**41**	**38**	**14**
***Examiner* II**	**89**	**92**	**70**	**68**	**35**	**66**	**72**	**39**	**40**	**16**
***Examiner* III**	**88**	**89**	**71**	**67**	**36**	**70**	**69**	**38**	**39**	**15**

(a) Calculate the Spearman's correlation coefficient of the marks awarded by:

(i) Examiners I and II;

(ii) Examiners I and III;

(iii) Examiners II and II.

(b) Use your results in (a) to determine which of the examiners agree most.

Candidates were required to calculate the spearman's rank correlation coefficient of the marks awarded by some examiners and conclude which examiners agreed most.

(a) Very few of the candidates attempted this question. The performance of the candidates was very poor. Most of the candidates could not do the ranking and squaring to be used in the formula. Most of them could not write the formular correctly and therefore mixed up the calculations.

Those who managed to solve the problem could not use their results to conclude which of the examiners agreed most.

Candidates were required to answer the question as follows:

13.(a)

Student	R_I	R_{II}	R_{III}	$d^2_{I,II}=(R_I - R_{II})^2$	$d^2_{I,\,III} = (R_I - R_{III})^2$	$d^2_{II,III} = (R_{II} - R_{III})^2$
A	1	2	2	1	1	0
B	2	1	1	1	1	0
C	4	4	3	0	1	1
D	6	5	6	1	0	1
E	9	9	9	0	0	0
F	3	6	4	9	1	4
G	5	3	5	4	0	4
H	7	8	8	1	1	0
I	8	7	7	1	1	0
J	10	10	10	0	0	0
				$\sum d^2_{I,\,II} = 18$	$\sum d^2_{I,\,III} = 6$	$\sum d^2_{II,\,III} = 10$

$$r_{12} = 1 - \frac{6\sum di^2}{n(n^2-1)}$$

$$1 - \frac{6 \times 18}{10 \times 99} = \frac{49}{55} = 0.8909$$

$$r_{13} = 1 - \frac{6 \times 6}{10 \times 99} = \frac{53}{55} = 0.9636$$

$$r_{23} = 1 - \frac{6 \times 10}{10 \times 99} = \frac{31}{33} = 0.9394$$

Examiners I and III agree most

(b) Examiners I and III agree most

QUESTION 14

The ends X and Y of an inextensible string 27 m long are fixed at two points on the same horizontal line which are 20 m apart. A particle of mass 7.5 kg is suspended from a point p on the string 12 m from X

(a) Illustrate this information in a diagram.

(b) Calculate, correct to two decimal places, $\angle YXP$ and $\angle XYP$.

(c) Find, correct to the nearest hundredth, the magnitudes of the tensions in the string.

[Take $g = 10\ m\ s^{-2}$]

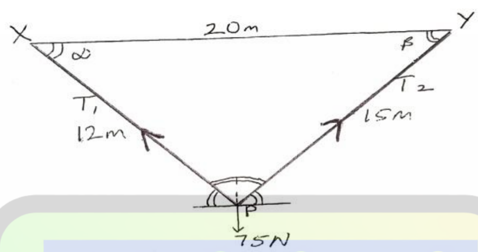
Candidates were required to illustrate forces in a diagram, use it to calculate angles and then to find the tensional forces in the string.

Most candidates could not draw the diagram and use it to solve the problem. Also, most candidates could not use the cosine and sine rules to solve the problem.

Candidates' performance was average.

Candidates were required to answer the question as follows:

14.(a)



$$\cos\alpha = \frac{20^2 + 12^2 - 15^2}{(2)(20)(12)}$$

$$= \frac{400+144-225}{480} = \frac{319}{480} = 0.6646$$

$$\alpha = \cos^{-1}0.6646 = 48.35^0$$

$$\cos\beta = \frac{20^2 + 15^2 - 12^2}{(2)(20)(15)}$$

$$= \frac{400+225-144}{600} = \frac{481}{600} = 0.8017$$

$$\beta = \cos^{-1}0.8017 = 36.71^0$$

(b) Using Lami's theorem

$$\frac{T_1}{\sin 126.71^0} = \frac{T_2}{\sin 138.35^0} = \frac{75}{\sin 94.94^0}$$

$$\frac{T_1}{\sin 126.71^0} = \frac{75}{\sin 94.94^0}$$

$$T_1 = \frac{75\times \sin 126.71^0}{\sin 94.94^0} = \frac{75\times 0.8017}{0.9963}$$

$$T_1 = 60.3495N$$

$$= 60.35N$$

(c)

$$\frac{T_2}{\sin 138.35^0} = \frac{75}{\sin 94.94^0}$$

$$T_2 = \frac{75 \times sin138.35^0}{sin94.94^0} = \frac{75 \times 0.6646}{0.9963}$$

$$T_2 = 50.0301N$$

$$= 50.03N$$

QUESTION 15

A particle P moves in a plane such that at time t seconds, its velocity, $v = (2ti - t^3j)\ m\,s^{-1}$.

(a) Find, when $t = 2$, the magnitude of the:

(i) velocity of P.

(ii) acceleration of P.

(b) Given that P is at the point with position vector $(3i + 2j)$ when $t = 1$, find the position vector of P when $t = 2$.

Candidates were required to use differentiation and integration of vectors (in i, j, k) to solve the problem. Most of the candidates scored good marks in the part (a). the part (b) was poorly done as a good number of the candidates could not integrate the velocity vector to obtain the position vector.

Candidates were required to answer the question as follows:

(a)(i)

$$v = 2ti - t^3j$$

When $t = 2,\ v = 4i - 8j$

$$|v| = \sqrt{4^2 + (-8)^2} = \sqrt{80}$$

$$= 4\sqrt{5}ms^{-1} or\ 8.944ms^{-1}$$

(ii)

$$a = 2 - 3t^2$$

$$a = 2i - 3t^2j$$

$$= 2i - 12j$$

$$|a| = \sqrt{2^2 + (-12)^2} = \sqrt{148}$$

$$= 2\sqrt{37}ms^{-2}\ or\ 12.166ms^{-2}$$

(b)

$$S = \frac{2t^2}{2}i - \frac{t^4}{4}j$$

$$S = t^2 - \frac{t^4}{4} + k$$

When t = 1, $(3i + 2j) = 1i - \frac{1}{4}j + k$

$$3i - i + 2j + \frac{1}{4}j = k$$

$$\left(2i + \frac{9}{4}j\right) = k$$

$$S = t^2i - \frac{t^4}{4}j + 2i + \frac{9}{4}j$$

$$= (t^2 + 2)i + \left(\frac{9}{4} - \frac{t^4}{4}\right)j$$

When t = 2, $S = 6i + \left(\frac{9}{4} - 4\right)j$

$$S = 6i + -\frac{7}{4}j$$

$$S = 6i - \frac{7}{4}j$$